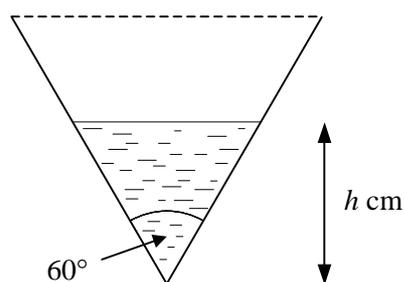


## DIFFERENTIATION

- 1 Given that  $y = x^2 + 3x + 5$ ,  
and that  $x = (t - 4)^3$ ,
- a find expressions for
- i  $\frac{dy}{dx}$  in terms of  $x$ ,      ii  $\frac{dx}{dt}$  in terms of  $t$ ,
- b find the value of  $\frac{dy}{dt}$  when
- i  $t = 5$ ,                              ii  $x = 8$ .
- 2 The variables  $x$  and  $y$  are related by the equation  $y = x\sqrt{2x-3}$ .  
Given that  $x$  is increasing at the rate of 0.3 units per second when  $x = 6$ , find the rate at which  $y$  is increasing at this instant.
- 3 The radius of a circle is increasing at a constant rate of  $0.2 \text{ cm s}^{-1}$ .
- a Show that the perimeter of the circle is increasing at the rate of  $0.4\pi \text{ cm s}^{-1}$ .
- b Find the rate at which the area of the circle is increasing when the radius is 10 cm.
- c Find the radius of the circle when its area is increasing at the rate of  $20 \text{ cm}^2 \text{ s}^{-1}$ .
- 4 The area of a circle is decreasing at a constant rate of  $0.5 \text{ cm}^2 \text{ s}^{-1}$ .
- a Find the rate at which the radius of the circle is decreasing when the radius is 8 cm.
- b Find the rate at which the perimeter of the circle is decreasing when the radius is 8 cm.
- 5 The volume of a cube is increasing at a constant rate of  $3.5 \text{ cm}^3 \text{ s}^{-1}$ . Find
- a the rate at which the length of one side of the cube is increasing when the volume is  $200 \text{ cm}^3$ ,
- b the volume of the cube when the length of one side is increasing at the rate of  $2 \text{ mm s}^{-1}$ .

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The diagram shows the cross-section of a right-circular paper cone being used as a filter funnel. The volume of liquid in the funnel is  $V \text{ cm}^3$  when the depth of the liquid is  $h \text{ cm}$ .

Given that the angle between the sides of the funnel in the cross-section is  $60^\circ$  as shown,

a show that  $V = \frac{1}{9}\pi h^3$ .

Given also that at time  $t$  seconds after liquid is put in the funnel

$$V = 600e^{-0.0005t},$$

- b show that after two minutes, the depth of liquid in the funnel is approximately 11.7 cm,
- c find the rate at which the depth of liquid is decreasing after two minutes.